Bivariate tail risk analysis for high-frequency returns via extreme value theory: supplementary material

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Abstract

This document summarizes extended analyses of tail risk in high-frequency financial portfolios, comparing an extreme value approach to a bivariate Gaussian model. Here, we examine one-minute and five-minute fluctuations in addition to the 15-second movements presented in the paper.

1 One-minute trading data

We examine security log-return data evaluated at one-minute intervals. Fitted intraday volatility curves are displayed in Figure 1, and the autocorrelation functions of volatility

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sequences before and after detrending are displayed in Figure 2 for JPM. We carry out the estimation and model comparison exercise described in Section 4 of the main document, which we summarize briefly here. As found with 15-second returns, the Gaussian approach generally underestimates the risk in both the upper and lower tails of the portfolios. Figure 3 displays the counts of exceedances of the time-varying VaR measure for each approach. Finally, Figure 4 shows the expected shortfall values predicted by the model compared to the actual shortfall values of the exceedances.

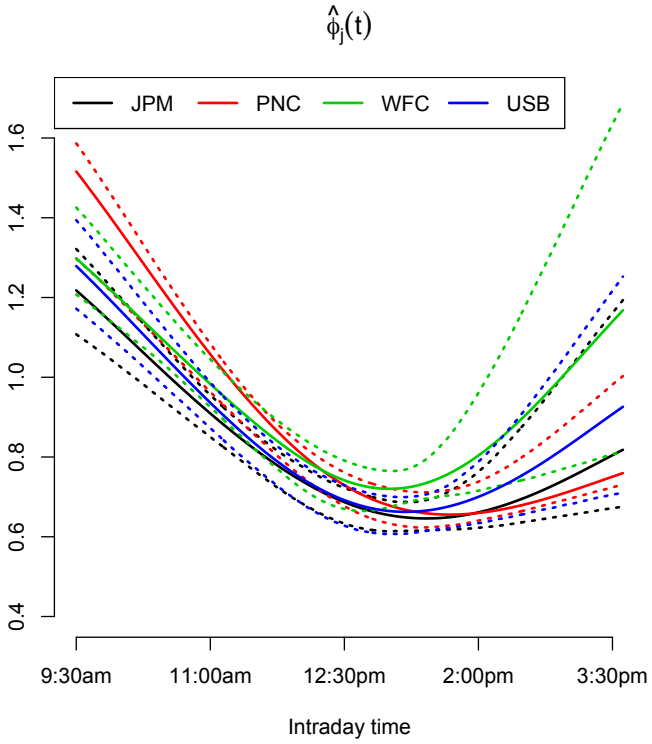

Fig. 1: Estimated smoothing spline fit with 95% confidence bands for intraday volatility trend using training data (one-minute intervals). Uncertainty bands obtained via bootstrapping volatility sequences.

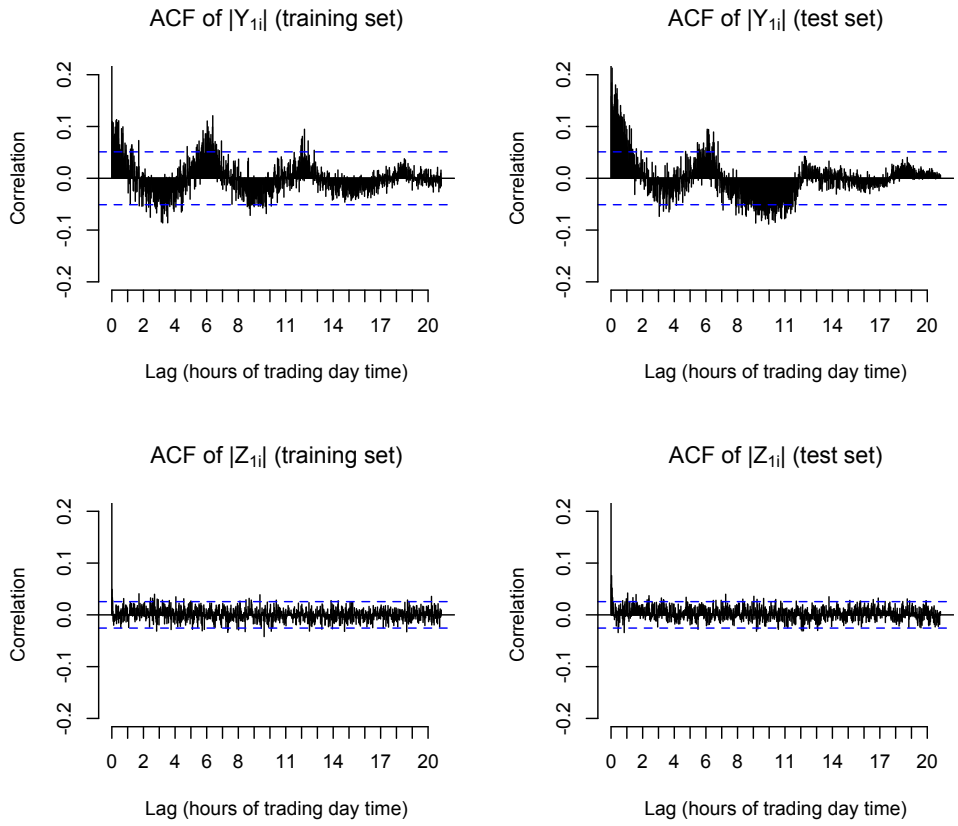

Fig. 2: ACF of absolute value of one-minute log-return of security JPM. The other stocks have similar patterns. Top left: original log-returns on training data. Top right: original log-returns on test data. Bottom left: residuals from spline smoothing and GARCH(1,1) on the training data. Bottom right: residuals from spline smoothing and GARCH(1,1) on the test data.

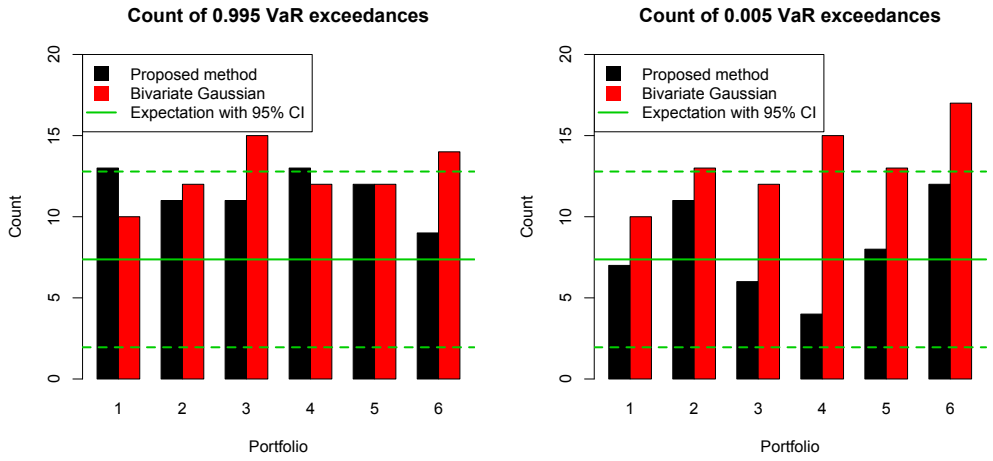

Fig. 3: Number of exceedances of portfolio 0.005 VaR and 0.995 VaR of extreme value model and bivariate Gaussian model on test data of one-minute log-returns. Each pair of bars corresponds to one pair of stocks. 1: (JPM,PNC), 2: (JPM,WFC), 3: (JPM,USB), 4: (PNC,WFC), 5: (PNC,USB), 6: (WFC,USB).

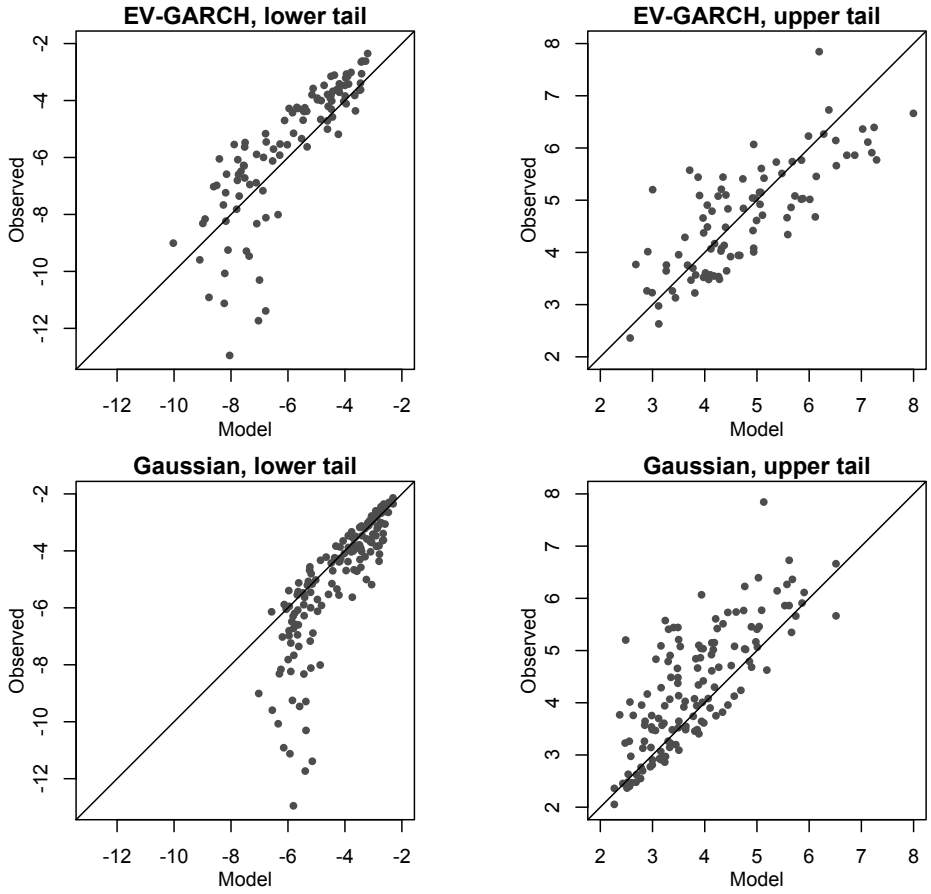

Fig. 4: Model-predicted portfolio expected loss (gain) for events with tail probability of less than 0.005 of one-minute log-return values (x-axis) against observed exceedances of predicted VaR values, for EVT (top) and Gaussian (bottom) methods and lower (left) and upper (right) tails.

2 Five-minute trading data

We repeat the same analyses described above on five-minute trading data. Figures 5–8 display the results for the five-minute returns. We see that the intraday pattern becomes more difficult to discern at five-minute granularity; as a result, the performance of both approaches is diminished.

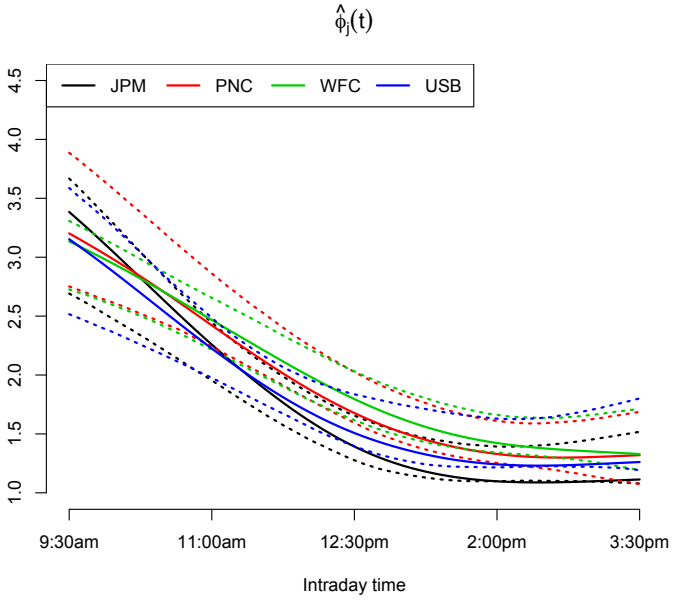

Fig. 5: Estimated smoothing spline fit with 95% confidence bands for intraday volatility trend using training data (five-minute intervals). Uncertainty bands obtained via bootstrapping volatility sequences.

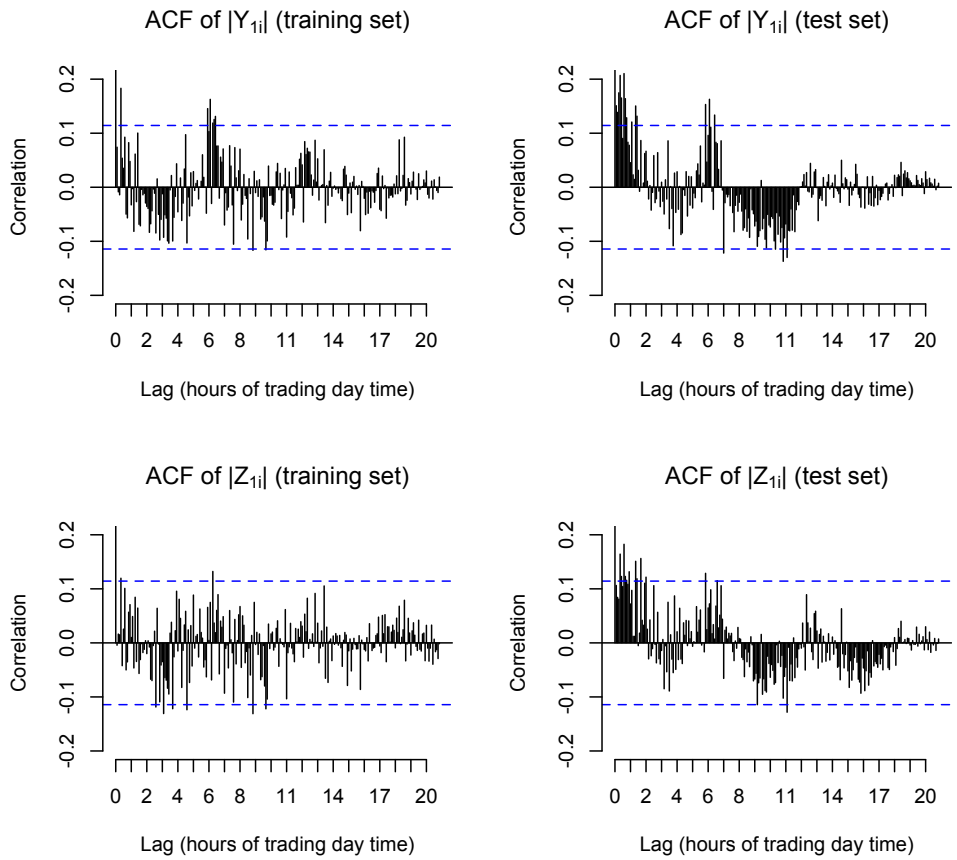

Fig. 6: ACF of absolute value of five-minute log-return of security JPM. The other stocks have similar patterns. Top left: original log-returns on training data. Top right: original log-returns on test data. Bottom left: residuals from spline smoothing and GARCH(1,1) on the training data. Bottom right: residuals from spline smoothing and GARCH(1,1) on the test data.

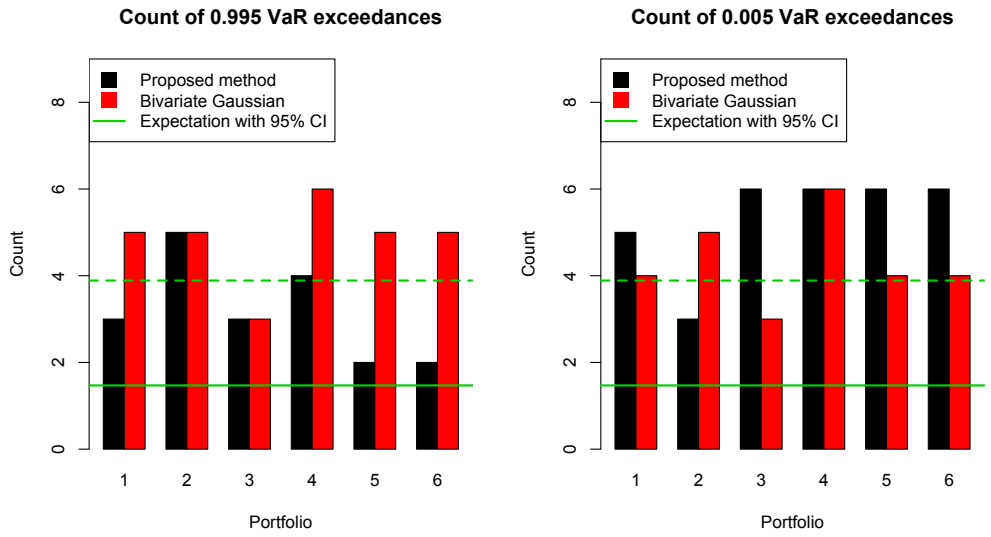

Fig. 7: Number of exceedances of portfolio 0.005 VaR and 0.995 VaR of extreme value model and bivariate Gaussian model on test data of five-minute log-returns. Each pair of bars corresponds to one pair of stocks. 1: (JPM,PNC), 2: (JPM,WFC), 3: (JPM,USB), 4: (PNC,WFC), 5: (PNC,USB), 6: (WFC,USB).

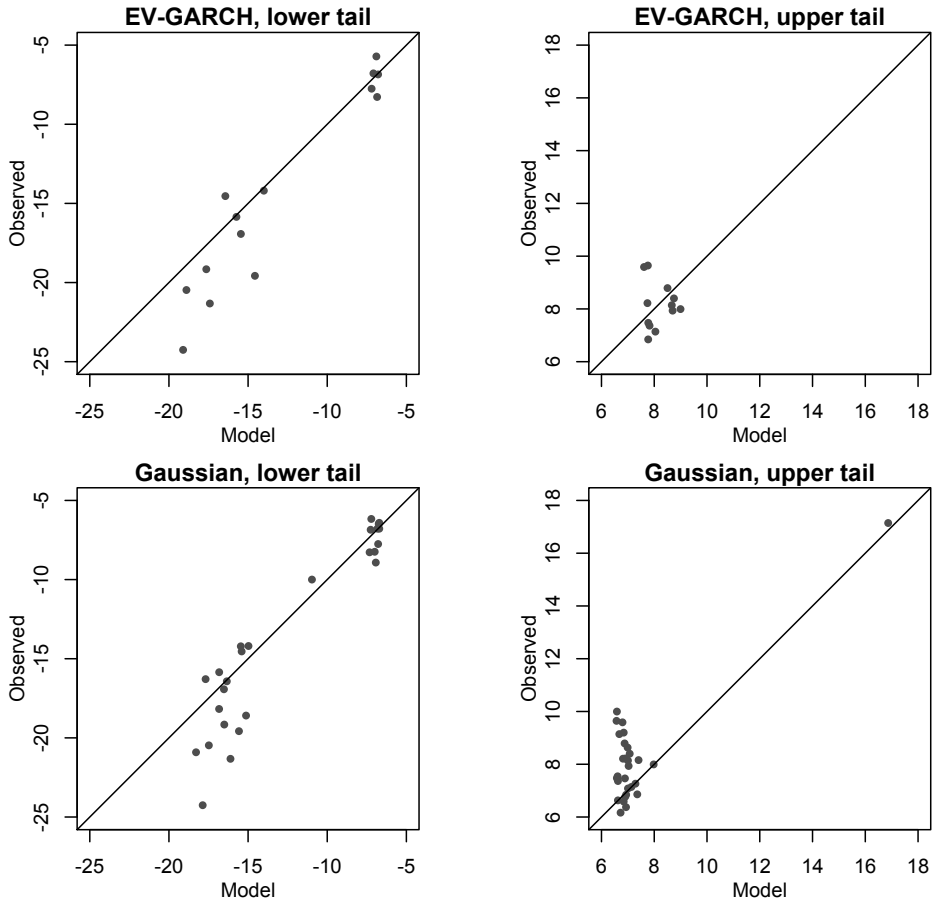

Fig. 8: Model-predicted portfolio expected loss (gain) for events with tail probability of less than 0.005 of five-minute log-return values (x-axis) against observed exceedances of predicted VaR values, for EVT (top) and Gaussian (bottom) methods and lower (left) and upper (right) tails.